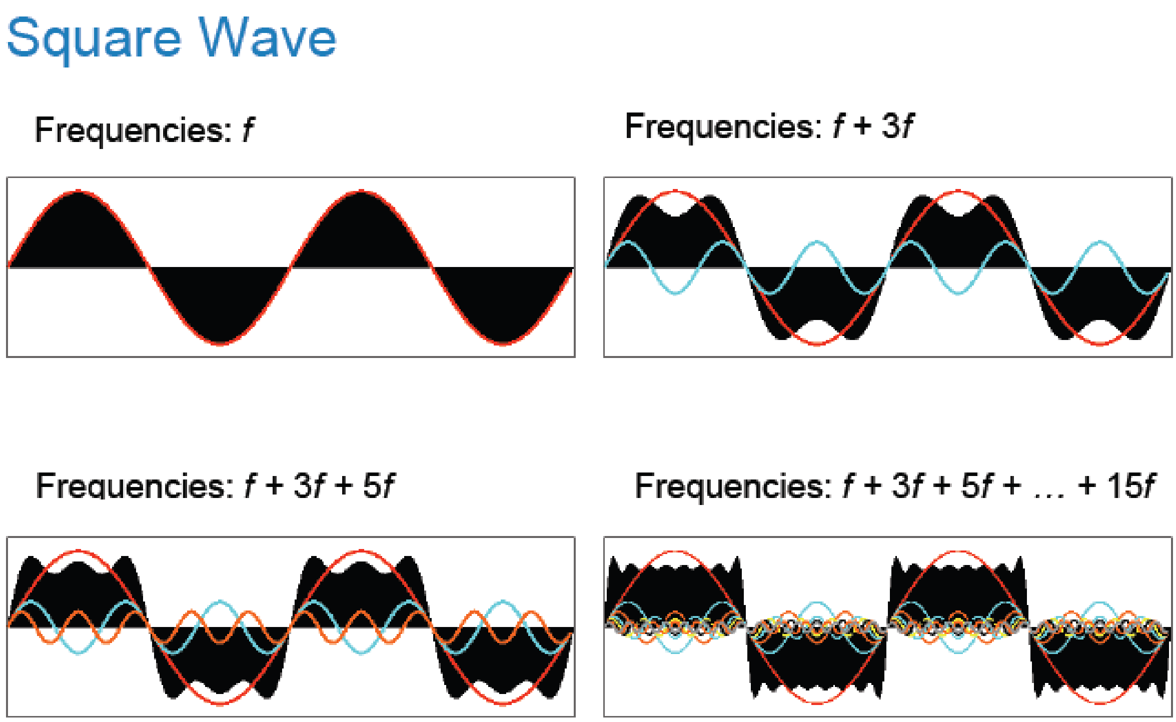
Data Communications

Lab Work 3

Fourier Series Representation of Square Waves

Any periodic signal can be represented by an infinite number of the sine and cosine harmonics of its fundamental frequency. However, in real life it is not possible to have infinitely many harmonics transmitted over a communication system. The following figure depicts square waveform using different number of harmonics



Write a MATLAB function to get the number of harmonics as input and draw the waveform of the square wave.

Appendix

Let be $f(x)$ a periodic function that is integrable on $[-\pi,\pi]$. Then, the Fourier coefficients of f are given by:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx\,, \qquad n \geq 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx\,, \qquad n \geq 1$$

The Fourier series of f is given by:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(nx) + b_n \sin(nx)\right]$$

The partial sum of the Fourier series for f with N harmonics is given by:

$$(S_N f)(x) = \frac{a_0}{2} + \sum_{n=1}^{N}\left[a_n \cos(nx) + b_n \sin(nx)\right], \qquad N \geq 0\,.$$

The Fourier series of square wave with cycle frequency f is given by:

$$x_{\text{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi(2k-1)ft)}{(2k-1)}$$

$$= \frac{4}{\pi}\left(\sin(2\pi.ft) + \frac{1}{3}\sin(6\pi.ft) + \frac{1}{5}\sin(10\pi.ft) + \ldots\right)$$

The Fourier series of square wave with angular frequency ω is given by:

$$x_{\text{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin((2k-1)\omega t)}{(2k-1)}$$

$$= \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \ldots\right)$$